The semester A examination for Honors Precalculus consists of two parts. Part 1 is selected response on which a calculator will not be allowed. Part 2 is short answer on which a calculator will be allowed.

Pages with the  symbol indicate that a student should be prepared to complete questions like these with or without a calculator.

The formulas below are provided in the examination booklet.

Trigonometric Identities:

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta = 1 + \tan^2\theta$

$\csc^2\theta = 1 + \cot^2\theta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

$\sin\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

$\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

$\tan\left(\dfrac{\theta}{2}\right) = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

Law of Cosines and Sines:

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Area of a Triangle:

Area $\Delta = \dfrac{1}{2}ab\sin C$

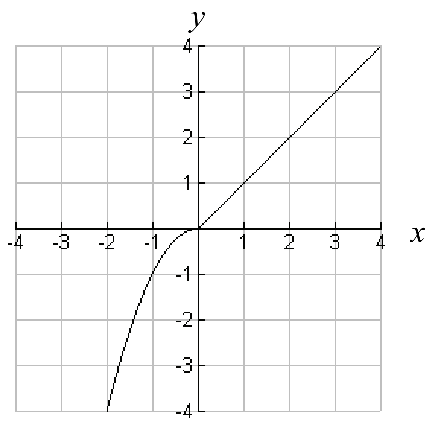
Area $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{1}{2}(a+b+c)$

Length of Arc:	**Linear Velocity:**
$s = r\theta$, θ in radians $s = \dfrac{\theta}{360}(2\pi r)$, θ in degrees	$v = r\omega$

PART 1 NO CALCULATOR SECTION

1. Sketch the graph of the piece-wise function $f(x)=\begin{cases} x^2\ , & \text{if } x<0 \\ \sqrt{x}+1, & \text{if } x\geq 0 \end{cases}$

2. Look at the graph of the piecewise function below.



Which of the following functions is represented by the graph?

A $f(x)=\begin{cases} x^2, & \text{if } x\leq 0 \\ x, & \text{if } x>0 \end{cases}$

B $f(x)=\begin{cases} -x^2, & \text{if } x\leq 0 \\ x, & \text{if } x>0 \end{cases}$

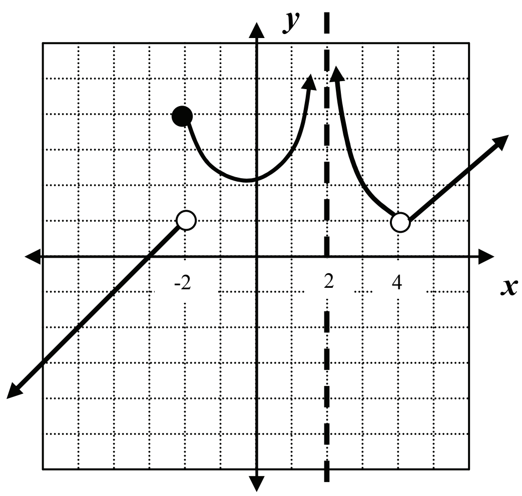
C $f(x)=\begin{cases} x, & \text{if } x\leq 0 \\ -x^2, & \text{if } x>0 \end{cases}$

D $f(x)=\begin{cases} -x, & \text{if } x\leq 0 \\ x^2, & \text{if } x>0 \end{cases}$

3. Sketch the graph of the following piece-wise function. Be sure to indicate arrows, closed circle endpoints, and open circle endpoints.

$$f(x)=\begin{cases}4, & \text{if } x<-2\\ x^2-3, & \text{if } -2\le x<3\\ 5-x, & \text{if } 3\le x\le 7\end{cases}$$

4. Look at the graph of the piecewise function below.



What type of discontinuity does the graph have at the following x values?

a. $x=-2$

b. $x=2$

c. $x=4$

5. Suppose that $\lim_{x\to 4^-} f(x)=8$, and that $f(4)$ is undefined.

a. Write a limit statement so that there is a removable discontinuity at $x=4$.

b. Write a limit statement so that there is a jump discontinuity at $x=4$.

6. Which of the following is true about the function $f(x)=\dfrac{x+4}{x-3}$?

A The function is continuous for all real numbers.

B The function is discontinuous at $x=3$ only.

C The function is discontinuous at $x=-4$ only.

D The function is discontinuous at $x=3$ and $x=-4$.

7. Determine whether each function below is even, odd, or neither even nor odd.

a. $g(x)=\sin x+x^3$

b. $h(x)=x^2-4$

c. $r(x)=\cos x+x^2$

d. $f\left(x\right)=x^2\cos x-x\sin x+4$

8. If $f(x)=x^{\frac{2}{3}}$, which of the following statements is NOT true?

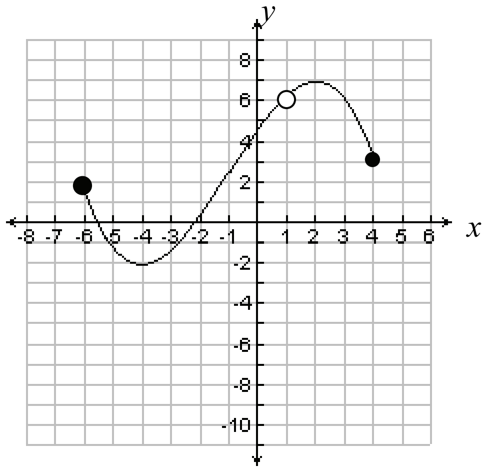
A The graph of $f(x)$ is symmetric with respect to the y-axis.

B $f(x)$ is an even function.

C The range of $f(x)$ is all real numbers.

D As $x\to\infty,\ f(x)\to\infty$

9. Look at the graph of the function below.



a. What is the domain of this function? ________________________

b. What is the range of this function? __________________________

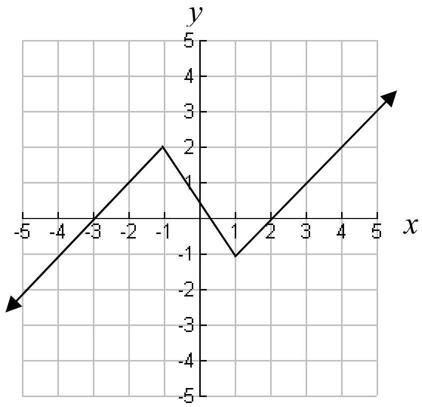
10. For each function below, find a formula for $f^{-1}(x)$ and state any restrictions on the domain.

a. $f(x)=\sqrt{x+2}$

b. $f(x)=x^3+4$

c. $f(x)=\dfrac{x+4}{x-2}$

11. Look at the graph of $f(x)$ below.



a. Sketch the graph of $f(|x|)$.

b. Sketch the graph of $f(2x)$

12. Match the transformations that would create the graph of $g(x)$ from the graph of $f(x)$.

_______ $g(x) = 3f(x)$ **A** Stretch the graph of $f(x)$ horizontally.

_______ $g(x) = f(3x)$ **B** Stretch the graph of $f(x)$ vertically.

_______ $g(x) = f\left(\frac{1}{3}x\right)$ **C** Shrink the graph of $f(x)$ horizontally.

_______ $g(x) = \frac{1}{3}f(x)$ **D** Shrink the graph of $f(x)$ vertically.

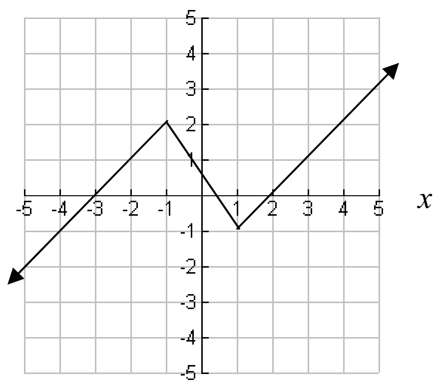
13. True or false.

a. The function $g(x) = 5f(x) - 2$ represents a vertical stretch of the graph of $f(x)$ by a factor of 5, followed by a vertical translation down 2 units.

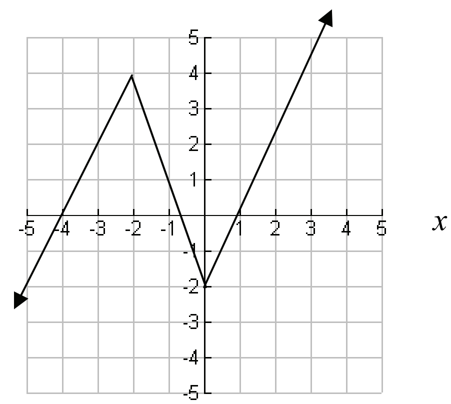
b. The function $g(x) = 7f\left(\frac{x}{4}\right)$ represents a vertical and horizontal shrinking of the graph of $f(x)$.

Problems 14 and 15 use the graphs of $f(x)$ and $g(x)$ below.

$y = f(x)$



$y = g(x)$



14. Which of the following represents the relationship between $f(x)$ and $g(x)$?

A $g(x) = 2f(x+1)$

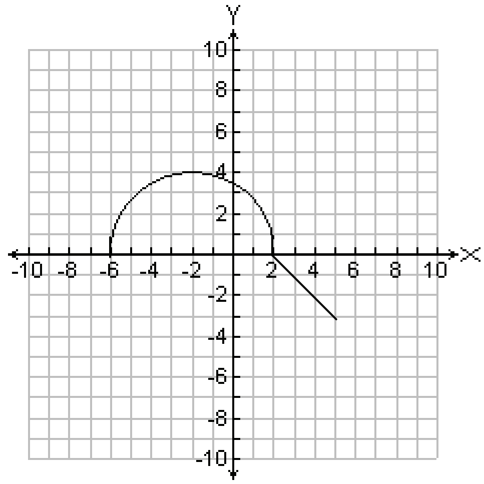
B $g(x) = \frac{1}{2}f(x-1)$

C $g(x) = f(2x) - 1$

D $g(x) = f\left(\frac{1}{2}x\right) + 1$

15. Sketch the graph of $y = |f(x)|$.

16. Look at the graph of $f(x)$ below. The domain of $f(x)$ is $-6 \le x \le 5$.



a. Describe how to transform the graph of $f(x)$ to $g(x) = 2f(x-1) - 3$

b. Sketch the graph of $g(x)$.

c. What is the domain of $g(x)$?

d. What is the range of $g(x)$?

17. Which of the following describes the end behavior of the function $f(x) = \dfrac{3}{x^2}$?

A $\lim_{x \to \infty} f(x) = -\infty$

B $\lim_{x \to \infty} f(x) = 0$

C $\lim_{x \to \infty} f(x) = 3$

D $\lim_{x \to \infty} f(x) = \infty$

18. Let x and y be related by the equation $x + y^2 = 17$.

   a. Determine the values of y if $x = 0$, 1, and 8.

   b. Is this relation a function? Justify your answer.

   c. Determine two functions defined implicitly by the relation.

19. Write the definitions of the six circular functions in terms of $x, y,$ and r.

20. If $\sin\theta = -\dfrac{4}{5}$ with $\cos\theta > 0$, what are the values of the other five trigonometric functions?

21. For each of the following, write the quadrant in which the terminal side of θ lies.

   a. $\sin\theta > 0, \tan\theta < 0$

   b. $\cos\theta < 0, \tan\theta > 0$

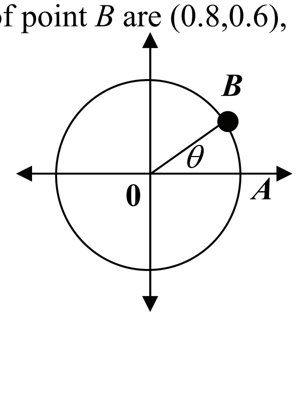
   c. $\sec\theta < 0, \csc\theta < 0$

22. Convert to radian measure. Leave your answer in terms of π.

   a. 40^o

   b. 165^o

23. The coordinates of point A are (1,0) and the coordinates of point B are (0.8,0.6), as shown below. Find the value of the following.



   a. $\sin\theta$

   b. $\cos\theta$

   c. $\tan\theta$

24. Determine the exact value of the following.

a.	$\sin\left(\frac{\pi}{6}\right)$	b.	$\cos\left(\frac{5\pi}{4}\right)$	c.	$\tan\left(\frac{5\pi}{3}\right)$
d.	$\sin\left(\frac{3\pi}{2}\right)$	e.	$\cos(\pi)$	f.	$\tan\left(\frac{\pi}{2}\right)$
g.	$\tan\left(-\frac{7\pi}{4}\right)$	h.	$\cos\left(-\frac{4\pi}{3}\right)$	i.	$\sin\left(-\frac{11\pi}{6}\right)$
j	$\sin\left(\frac{\pi}{4}\right)$	k.	$\cos\left(\frac{5\pi}{6}\right)$	l.	$\tan\left(\frac{7\pi}{6}\right)$
m.	$\sec\left(-\frac{5\pi}{4}\right)$	n.	$\cot\left(\frac{5\pi}{6}\right)$	o.	$\csc\left(\frac{4\pi}{3}\right)$

25. Given $\overset{\frown}{AB}$ on a unit circle with $A(1,0)$ and $B\left(-\frac{5}{13},\frac{12}{13}\right)$, determine the value of the following:

a. $\sin\overset{\frown}{AB}$

b. $\cos\overset{\frown}{AB}$

c. $\tan\overset{\frown}{AB}$

26. Which of the following are equal to $\cos 125^o$? Write yes or no for each.

a.	$\cos 145^o$	g.	$\cos 295^o$
b.	$\sin 145^o$	h.	$\frac{1}{\sec 125^o}$
c.	$\cos 215^o$	i.	$\cos 325^o$
d.	$\sin 215^o$	j.	$\sin 325^o$
e.	$\cos 235^o$		
f.	$\sin 235^o$		

27. Sketch the graphs of the six circular functions on the interval $-2\pi \le x \le 2\pi$

28. Which of the following functions does NOT have a period of 2π ?

A $f(x) = \sin x$

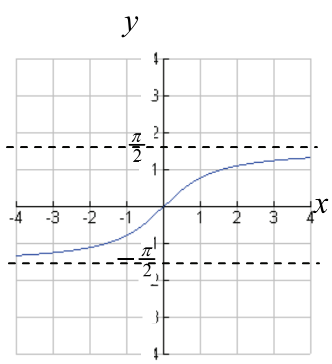
B $f(x) = \tan x$

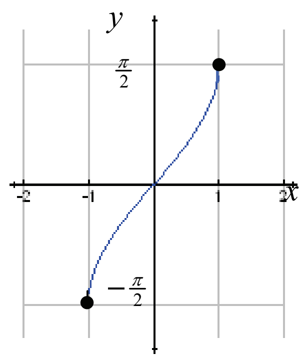
C $f(x) = \sec x$

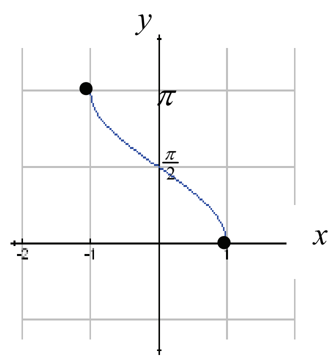
D $f(x) = \csc x$

29. a. Write an equation for each inverse function graph (i-iii).

i. ____________ ii. ____________ iii. ____________







b. Use the following intervals to complete the table below.

[-1,1] $(-\infty, \infty)$ $[0, \pi]$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

	$\text{Sin}^{-1}x$	$\text{Cos}^{-1}x$	$\text{Tan}^{-1}x$
Domain			
Range			

30. In terms of limits, describe the end behaviors of $f(x) = \text{Tan}^{-1}x$.

31. Determine the exact value of the following.

a. $\text{Sin}^{-1}\left(\frac{1}{2}\right)$

b. $\text{Cos}^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

c. $\text{Tan}^{-1}\left(\sqrt{3}\right)$

d. $\text{Sin}^{-1}(-1)$

e. $\text{Cos}^{-1}(0)$

f. $\text{Tan}^{-1}(-1)$

32. Determine the exact value of the following.

a. $\cos\left(\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

b. $\sin\left(\text{Tan}^{-1}(-1)\right)$

c. $\tan\left(\text{Cos}^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right)$

33. Determine the exact value of the following.

a. $\sin\left(\text{Csc}^{-1}\left(\frac{8}{5}\right)\right)$

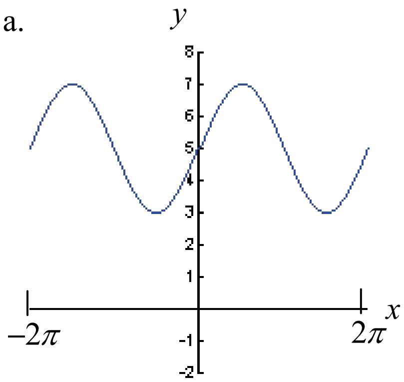
b. $\tan\left(\text{Sin}^{-1}\left(\frac{12}{13}\right)\right)$

c. $\text{Cos}^{-1}\left(\cos\left(\frac{11\pi}{6}\right)\right)$

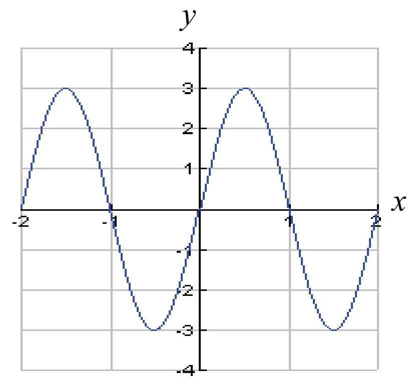
d. $\sin\left(\text{Tan}^{-1}\left(-\frac{4}{3}\right)\right)$

34. Write the sinusoidal equation for the following graphs

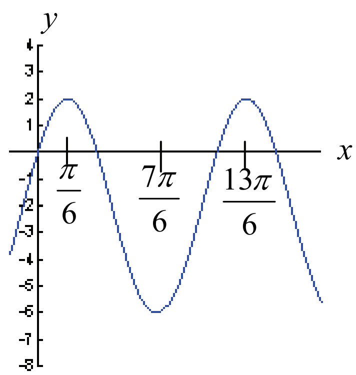
a.



b.



c.



35. Determine the equation that best describes a sine curve with amplitude 3, period of 6, and a phase shift of $\frac{\pi}{2}$ to the right.

36. State the amplitude, period, the phase shift and vertical translation of the sinusoid relative to the basic function $f(x)=\sin x$ or $f(x)=\cos x$. Sketch the graph, marking the *x*- and *y*-axes appropriately.

a. $f(x)=2\sin\left(3\left(x-\frac{\pi}{6}\right)\right)+5$

b. $f(x)=-5\cos\left(\pi\left(x+1\right)\right)$

c. $f(x)=5\sin\left(4x-\pi\right)-2$.

37. Assume that $\angle A + \angle B = 180^o$. Which of the following is equal to $\cos A$?

A $\sin A$
B $-\sin A$
C $\cos B$
D $-\cos B$

38. Simplify the following expressions and evaluate.

a. $\sin\frac{5\pi}{8}\cos\frac{3\pi}{8} - \cos\frac{5\pi}{8}\sin\frac{3\pi}{8}$

b. $\cos\frac{5\pi}{6}\cos\frac{\pi}{6} - \sin\frac{5\pi}{6}\sin\frac{\pi}{6}$

39. If $\sin A = \frac{5}{13}$ and $\cos A < 0$, determine

a. $\sin 2A$

b. $\cos 2A$

40. Suppose that $\cos\theta = -\frac{8}{17}, \sin\theta < 0$

Determine the exact values of each of the following.

a. $\sin\left(\frac{\theta}{2}\right)$

b. $\cos\left(\frac{\theta}{2}\right)$

c. $\tan\left(\frac{\theta}{2}\right)$

41. Prove the following identities.

a. $\sin\theta\cot\theta = \cos\theta$

b. $(\sin x + \cos x)^2 = 1 + \sin 2x$

c. $\dfrac{\csc x}{1+\cot^2 x} = \sin x$

d. $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \sec\theta\csc\theta$

e. $\sin(x+y) + \sin(x-y) = 2\sin x\cos y$

f. $\sin^2\theta + \sin^2\theta\tan^2\theta = \tan^2\theta$

g. $\tan x + \cot x = 2\csc(2x)$

h. $\dfrac{\cot\theta}{\cos\theta} + \dfrac{\sec\theta}{\cot\theta} = \sec^2\theta\csc\theta$

42. Solve the following equations on the interval $0^o \le \theta < 360^o$.

a. $2\sin\theta = -\sqrt{2}$

b. $3\cos\theta + 4 = 5\cos\theta + 5$

43. Solve the following equations on the interval $0 \le x < 2\pi$

a. $\tan x + 1 = 0$

b. $2\sin^2 x - 3\sin x + 1 = 0$

c. $2\sin(2x) + 1 = 0$

44. How many triangles are possible, if $a = 40$, $c = 80$, and $\angle A = 30^o$?

PART 2 CALCULATOR SECTION

A calculator may be used on questions 45 through 64.

45. A wheel of radius 12 cm turns at 7 revolutions per second. Determine the linear velocity of a point on the edge of the wheel in meters per second.

46. A moon makes a circular revolution around its planet in 80 hours. The radius of its circular path is 20,000 miles.

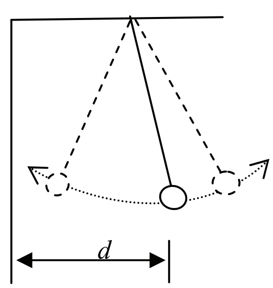
  a. What is the angular velocity of the planet, in radians per hour?

  b. What is the linear velocity of the planet, in feet per second?

47. Complete the following chart using $s = r\theta$.

Radius	Angle(radians)	Arc Length
6 inches	$\frac{\pi}{4}$	
	$\frac{5\pi}{6}$	15π feet
10 meters		30 meters

48. A ball on a string is swinging from the ceiling, as shown in the figure below.



Let d represent the distance that the ball is from the wall at time t.
Assume that d varies sinusoidally with time.

When $t = 0$ seconds, the ball is farthest from the wall, $d = 160$ cm.
When $t = 3$ seconds, the ball is closest to the wall, $d = 20$ cm.
When $t = 6$ seconds, the ball is again farthest from the wall, $d = 160$ cm.

  a. Sketch a graph of d as a function of time.

  b. Write a trigonometric function for d as a function of time.

  c. What is the distance of the ball from the wall at $t = 5$ seconds?

  d. What is the value of t the <u>first</u> time the ball is 40 cm from the wall?

49. Sara is riding a Ferris wheel. Her sister, Kari, starts a stopwatch and records some data. Let *h* represent Sara's height above the ground at time *t*. Kari notices that Sara is at the highest point, 80 feet above the ground, when $t = 3$ seconds. When $t = 7$ seconds Sara is at the lowest point, 20 feet above the ground. Assume that the height *h* varies sinusoidally with time *t*.

    a. Write a trigonometric equation for the height *h* of Sara above the ground as a function of time *t*.

    b. What will the height of Sara be above the ground at t = 11.5 seconds?

    c. Determine the first two times, $t > 0$, when the height of Sara above the ground is 70 feet.

50. At Ocean Tide Dock the first low tide of the day occurs at midnight, when the depth of the water is 2 meters, and the first high tide occurs at 6:30 A.M. with a depth of 8 meters. Assume that the depth of the water varies sinusoidally with time.

    a. Sketch and label a graph showing the depth (*d*) of the water at the dock as a function of time (*t*). Let $t = 0$ represent midnight.

    b. Determine a trigonometric model that represents the graph.

    c. Suppose a tanker requiring at least 3 meters of water depth is planning to dock after midnight. Determine the earliest possible time that the tanker can dock.

51. Solve for θ, where $0^o \le \theta < 360^o$.

    a. $3\cos\theta + 9 = 7$

    b. $3\sin^2\theta + 7\sin\theta + 2 = 0$

52. How many triangles *ABC* are possible if $\angle A = 20^o$, $b = 40$, and $a = 10$?

53. Given $\triangle ABC$, where $\angle A = 41^o$, $\angle B = 58^o$, and $c = 19.7$ cm, determine the measure of side *b*.

54. In ΔABC, $a = 9, b = 12, c = 16$. What is the measure of $\angle B$?

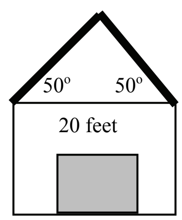
55. Determine the remaining sides and angles of a triangle with $\angle A = 58^{o}$, side a = 11.4 and side b = 12.8.

56. From a point 200 feet from its base, the angle of elevation from the ground to the top of a lighthouse is 55 degrees. How tall is the lighthouse?

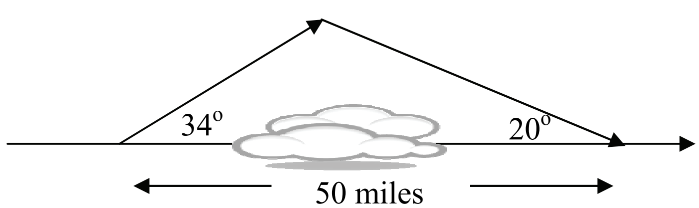
57. A truck is traveling down a mountain. A sign says that the degree of incline is 7 degrees. After the truck has traveled one mile (5280 feet), how many feet in elevation has the truck fallen?

58. A person is walking towards a mountain. At one point, the angle of elevation from the ground to the top of the mountain is 37 degrees. After walking another 1000 feet, the angle of elevation from the ground to the top of the mountain is 40 degrees. How high is the mountain?

59. The owner of the garage shown below plans to install a trim along the roof. The lengths required are in bold. How many feet of trim should be purchased?



60. An airplane needs to take a detour around a group of thunderstorms, as shown in the figure below. How much farther does the plane have to travel due to the detour?

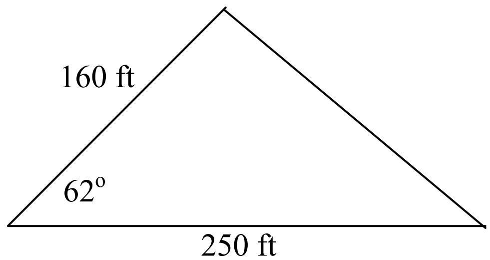


61. Determine the area of each triangle. Give answer to the nearest tenth if necessary.

    a. $\Delta ABC : a = 4, b = 10, m\angle C = 30^o$

    b. $\Delta ABC : a = 17, b = 13, c = 18$

62. A real estate appraiser wishes to find the value of the lot below.



    a. Find the area of the lot.

    b. An acre is 43560 square feet. If land is valued at $56,000 per acre, how much is the land worth?

63. Triangle ABC has an area of 2400 with $AB = 80, AC = 100$. Determine the two possible measures of angle A.

64. A regular pentagon is inscribed in a circle of radius 40 meters. What is the length of the apothem of the pentagon?