

Honors Precalculus B Exam Review

The semester B examinations for Precalculus/Honors Precalculus will consist of two parts. Part 1 will be selected response on which a calculator will not be allowed. Part 2 will be short answer on which a calculator will be allowed.

The following symbol applies to this review:



Indicates that a item like this may be on the no-calculator, selected response section **or** the calculator section of the exam.

Compound Interest Formulas:

Continuous compounding: $A(t) = Pe^{rt}$ compounded k times per year: $A(t) = P\left(1 + \frac{r}{k}\right)^{kt}$

Newton's Law of Cooling: $T(t) = T_m + (T_0 - T_m)e^{-kt}$

Parametric equations for projectile motion: $x = (v_0 \cos \theta)t$
 $y = -16t^2 + (v_0 \sin \theta)t + h_0$

Distance from the point (x_1, y_1) to the line $Ax + By + C = 0$: $\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

If $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$, then $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2$

If $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$, then $\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

If $\vec{u} = \langle u_1, u_2 \rangle$, then

$|\vec{u}| = \sqrt{u_1^2 + u_2^2}$ and the direction of $\vec{u} = \tan^{-1} \left| \frac{u_2}{u_1} \right|$, placed in the appropriate quadrant.

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The angle between \vec{u} and $\vec{v} = \text{Cos}^{-1}\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right)$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Sum of an infinite geometric series:

$$S = \frac{a_1}{1-r}, \text{ if } |r| < 1$$

Binomial Theorem:

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + \binom{n}{n}b^n$$

$$\text{where } \binom{n}{r} = nCr = \frac{n!}{r!(n-r)!}$$

OR

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{1\cdot 2}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}y^3 + \dots + y^n = \sum_{r=0}^n \frac{n!}{r!(n-r)!}x^{n-r}y^r$$

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Decimal approximations should be given to 3 places after the decimal point.

For items 1 through 3, sketch the graph of the function. Write the equations of any asymptotes, and the coordinates of any intercepts.



1. $f(x) = \frac{3}{x^2 - 3x - 10}$



2. $f(x) = \frac{x-4}{x^2 + 2x - 8}$



3. $f(x) = 2 + \frac{5}{x-3}$

For items 4 and 5, sketch the graph of the function. Write the equations of any asymptotes, the domain and range, and the x - and y -coordinates of any removable discontinuities, and the coordinates of any intercepts.



4. $f(x) = \frac{x^2 - 7x + 6}{x^2 - 36}$



5. $f(x) = \frac{x^2 - 8x + 7}{x - 6}$



6. Which of the following is the equation of the horizontal asymptote of the graph of the function $f(x) = \frac{4x^2 - 2}{x^2 - 5}$

A $x = \frac{2}{5}$

B $x = 5$

C $y = \frac{1}{2}$

D $y = 4$

For items 7 and 8, solve the equations. Check for any extraneous roots.



7. $\frac{3x}{x-1} = \frac{12}{x^2-1} + 2$



8. $\frac{x}{x-2} + \frac{3x}{x-4} = \frac{32-2x}{x^2-6x+8}$

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For items 9 and 10, solve the inequalities.



9. $\frac{x}{x-2} < 0$



10. $\frac{x-3}{x^2+7x+10} \geq 0$

For items 11 and 12, use partial fraction decomposition to write an expression equivalent to the given expression.

11. $\frac{13x-31}{x^2-5x+6}$

12. $\frac{-9}{x^2-x-2}$

13. $\lim_{x \rightarrow 2} \frac{x^2-5x+6}{x^2+8x-20} =$



14. Which of the following statements is false?

- A $f(x) = \ln x$ has a domain of all positive real numbers
- B $f(x) = \ln x$ increases without bound as x increases without bound.
- C The graph of $f(x) = e^{-x}$ is the graph of $g(x) = e^x$ reflected about the y -axis.
- D $f(x) = e^{-x}$ has a range of all real numbers.



15. Which of the following is equivalent to $\log_5 \left(\frac{5}{x^3} \right)$?

- A $5 - 3\log_5 x$
- B $1 - 3\log_5 x$
- C $-3\log_5 x$
- D $2\log_5 x$



16. Which of the following is equivalent to $\log\left(\frac{y^2}{\sqrt{x}}\right)$?

- A $2\log y - 2\log x$
- B $2\log y - \frac{1}{2}\log x$
- C $\log(2y) - \log\left(\frac{1}{2}x\right)$
- D $\frac{\log(y^2)}{\log(\sqrt{x})}$



17. Which of the following is equivalent to $3\ln x + \ln z - \ln y$?

- A $3\ln\left(\frac{xz}{y}\right)$
- B $\ln\left(\frac{xz}{y}\right)^3$
- C $\ln\left(\frac{x^3z}{y}\right)$
- D $\ln\left(\frac{(xz)^3}{y}\right)$



18. Which of the following is equivalent to $\frac{\log 20}{\log 2}$?

- A $\log 10$
- B $\log 18$
- C $\log_2 20$
- D $\log_{20} 2$

For items 19 and 20, evaluate. Write your answer correct to three decimal places.

19. $\log_7 6904$

20. $\log_4 0.0123$



21. The graph of $g(x)$ is the graph of $f(x) = 3^x$ reflected about the x -axis and translated 5 units to the left. What is the function rule for $g(x)$?



22. The graph of $g(x)$ is the graph of $f(x) = 9^x$ reflected about the y -axis and stretched horizontally by a factor of 7. What is the function rule for $g(x)$?

For items 23 through 30, solve. If necessary, write your answer correct to three places after the decimal point.



23. $3e^{2x} - 7 = 50$

24. $2.8^x = 345$

25. $5 \cdot 2^{3x-1} + 20 = 520$

26. $\ln(x + 4) = 2$

27. $\log_7(3x - 1) = 2$



28. $\log_5 x + \log_5(x - 4) = 1$



29. $\log_2(x + 5) - \log_2(x - 1) = 2\log_2 2$



30. $2 - \log x = \log 20$

31. A colony of insects has an initial population of 600. The number of insects triples every 4 weeks.

- Write a function for the number of insects after t weeks.
- What will the number of insects be after 7 weeks?
- After how many weeks will the number of insects be 12000?

32. A population that grows continuously has an initial value of 400. After $t = 8$ years, the population is 1000. What is the continuous growth rate of this population?

33. Jack puts \$5000 in the bank at 6% interest compounded monthly. How many years will it take the amount of money in the account to become \$6000 ?

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34. The population of a certain city t years after 2000 is given by the function $P(t) = 1300e^{0.04t}$.
- What was the initial population in 2000?
 - What is the rate of growth of the population?
 - What will the population be in 2010?
 - At what time t will the population be 2500?
35. The percentage of adult height obtained by a boy x years old is modeled by the function $f(x) = 29 + 49 \log(x+1)$.



- Approximately what percentage of his adult height is a 9 year-old?
- At what age will a boy reach 90% of his adult height?



36. For the function $g(x) = -\ln(x+2) - 5$,
- What are the domain and range?
 - What are the equations of any asymptotes?
 - Describe the transformations to the graph of $f(x) = \ln x$ that results in the graph of $g(x)$.

37. The logistic function $f(t) = \frac{30000}{1+19e^{-1.5t}}$ describes the number of people $f(t)$ that have become ill with the flu t weeks after its initial outbreak.
- How many people became ill when the outbreak first began?
 - After approximately how many weeks were there 10000 people ill?
 - What is the maximum number of people that can become ill?

38. A cup of hot chocolate with a temperature of 90°C is placed in a room that is 20°C . After 1 minute, the hot chocolate has cooled to a temperature of 82°C . How long will it take for the hot chocolate to cool to a temperature of 50°C ?

For items 39 and 40, eliminate the parameter to find the relationship between x and y .



39. $x = 3t$
 $y = 6t - 7$



40. $x = 2t + 4$
 $y = 4t + 5$



41. Eliminate the parameter to find the relationship between x and y :
$$\begin{aligned}x &= 5 \cos t \\y &= 8 \sin t\end{aligned}$$
42. A golfer hits the ball from the ground at a 20° angle with an initial velocity of 132 feet per second.
- Give the motion of the ball as a set of parametric equations representing the horizontal movement of the ball after t seconds.
 - Give the location of the ball after 2 seconds.
43. A punter kicks a football from at height of 2 feet with an initial velocity of 82 feet per second at an angle of 63 degrees with the horizontal.
- Write parametric equations for the horizontal and vertical distances that the ball is from the punter after t seconds.
 - What is the position of the ball at $t = 1.5$ seconds.
 - A player 6 feet tall is standing 50 yards (150 feet) from the punter directly in the path of the kick. Will the ball go over his head? Why or why not?
 - Suppose that the kick returner lets the ball hit the ground. When will the ball hit the ground, and how far from the punter will the ball hit the ground?



44. Write the vector and parametric equations of the line that passes through the points $(1, 4)$ and $(3, 9)$.

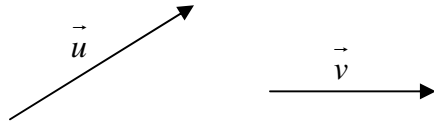


45. Given the points $A(2, -1)$ and $B(0, 4)$. What is the vector \overline{AB} in component form?
- A** $\langle 2, -5 \rangle$
B $\langle -2, 5 \rangle$
C $\langle 2, 3 \rangle$
D $\langle -2, -3 \rangle$

46. What is the magnitude and direction of the vector $\langle -6, 8 \rangle$?

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Look at the vectors \vec{u} and \vec{v} below



47. Represent the graphically the vector $\vec{u} + \vec{v}$ using both the head-to-tail method and the parallelogram method of addition.
48. Represent the vector $\vec{u} - \vec{v}$ graphically.

For items 49 through 52, use the vectors $\vec{a} = \langle 2, -3 \rangle$, $\vec{b} = \langle 6, 5 \rangle$, and $\vec{c} = \langle r, -9 \rangle$



49. $\vec{a} \cdot \vec{b} =$

50. The angle between \vec{a} and $\vec{b} =$



51. Find the value of r such that the vectors \vec{a} and \vec{c} are parallel.



52. Find the value of r such that the vectors \vec{b} and \vec{c} are perpendicular.

53. A vector in space has an initial point of $A(-2, -3, 4)$ and $B(-1, 4, -5)$.

- Write \overline{AB} in component form.
- What is the magnitude of \overline{AB} ?

54. Let vectors $\vec{u} = \langle -4, 1, 5 \rangle$ and $\vec{v} = \langle -3, 2, -1 \rangle$

- Determine the angle between the vectors.
- Determine the cross product of the vectors.
- Describe the geometric meaning of the cross product that you found in part (b) above.
- Write the vector equation of the line that is parallel to \vec{u} and passes through the point $P(2, 4, -3)$.

55. A plane is headed due west at 400 mph. A wind is blowing towards the northwest at 60 mph.

- What is the ground speed of the plane?
- In what direction is the plane traveling?

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For items 56 and 57, write the equivalent rectangular coordinates for each point in polar form.



56. $\left(8, \frac{\pi}{3}\right)$



57. $\left(-4, -\frac{7\pi}{4}\right)$

For items 58 and 59, write the equivalent polar form for each point in rectangular form.



58. $(4, -4\sqrt{3})$



59. $(-1, -1)$



60. Which of the following represents the same point as $\left(4, -\frac{\pi}{3}\right)$?

A $\left(4, \frac{\pi}{3}\right)$

B $\left(-4, \frac{2\pi}{3}\right)$

C $\left(4, \frac{2\pi}{3}\right)$

D $\left(-4, \frac{5\pi}{3}\right)$

For items 61 and 62, graph each equation.

61. $r = 1 - 2\cos \theta$

62. $r = 4\sin 2\theta$

For items 63 and 64, write each rectangular equation in polar form.



63. $x + y = 9$



64. $x = 7$

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For items 65 and 66, write each polar equation in rectangular form.



65. $r \sin \theta = 6$



66. $r = \frac{3}{2 \cos \theta + 5 \sin \theta}$

For items 67 and 68, write the complex number in polar form.



67. $-6\sqrt{3} - 6i$

68. $3 + i\sqrt{3}$

For items 69 and 70, write the complex number in rectangular form.



69. $6 \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right)$



70. $-2(\cos 7\pi + i \sin 7\pi)$

For items 71 and 72, simplify. Write your answers in polar form.



71.
$$\frac{\left[3 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \right] \left[4 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \right]}{6 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)}$$



72. $\left[3(\cos 55^\circ + i \sin 55^\circ) \right]^4$

For items 73 and 74, determine the coordinates (r, θ) that are solutions for each system, where.

73.
$$\begin{aligned} r &= 2 \cos \theta & 0 \leq \theta < 360^\circ \\ r &= 1 - 3 \cos^2 \theta \end{aligned}$$

74.
$$\begin{aligned} r &= \sin \theta & 0 \leq \theta < 2\pi \\ r &= \sin(2\theta) \end{aligned}$$



75. A plane has the equation $5x + 2y - 4z = 20$. What are the x -, y -, and z -intercepts of the plane?

For items 76 and 77, represent the series using summation notation.



76. $8 + 16 + 32 + 64 + \dots$



77. $11 + 15 + 19 + 23 + 27 + 31 + 35$

For items 78 through 80, determine the sum, if any, of the series.



78. $9 + 3 + 1 + \frac{1}{3} + \dots$



79. $\sum_{n=1}^{\infty} 3 \cdot 2^n$



80. $\sum_{n=1}^{\infty} 8 \left(\frac{3}{4}\right)^n$

For items 81 and 82, evaluate.



81. $\sum_{n=1}^4 (3n + 1)$



82. $\sum_{n=5}^8 4n$



83. What is the sum of the series $1 + x + x^2 + x^3 + x^4 + \dots$?



84. Expand. $(x - 2)^4$



85. What is the coefficient of the x^3 term in the expansion of $(x + 2)^5$?



86. What is the coefficient of the x^9y term in the expansion of $(x^3 + 5y)^4$?