

Precalculus B Exam Review

The semester B examination for Precalculus will consist of two parts. Part 1 will be selected response on which a calculator will not be allowed. Part 2 will be short answer on which a calculator will be allowed.

The following symbols apply to this review:



Indicates that a item like this may be on the no-calculator, selected response section **or** the calculator section of the exam.

Compound Interest Formulas:

Continuous compounding: $A(t) = Pe^{rt}$ compounded k times per year: $A(t) = P\left(1 + \frac{r}{k}\right)^{kt}$

Permutation ${}_nP_r = \frac{n!}{(n-r)!}$, Combination ${}_nC_r = \frac{n!}{r!(n-r)!}$

Parametric equations for projectile motion:
 $x = (v_0 \cos \theta)t$
 $y = -16t^2 + (v_0 \sin \theta)t + h_0$

Distance from the point (x_1, y_1) to the line $Ax + By + C = 0$: $\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

If $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$, then $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2$

If $\vec{u} = \langle u_1, u_2 \rangle$, then

$|\vec{u}| = \sqrt{u_1^2 + u_2^2}$ and the direction of $\vec{u} = \tan^{-1} \left| \frac{u_2}{u_1} \right|$, placed in the appropriate quadrant.

The angle between \vec{u} and $\vec{v} = \cos^{-1} \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \right)$

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Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Sum of an infinite geometric series:

$$S = \frac{a_1}{1-r}, \text{ if } |r| < 1$$

Binomial Theorem:

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + \binom{n}{n}b^n$$

$$\text{where } \binom{n}{r} = nCr = \frac{n!}{r!(n-r)!}$$

OR

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{1 \cdot 2}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}x^{n-3}y^3 + \dots + y^n = \sum_{r=0}^n \frac{n!}{r!(n-r)!}x^{n-r}y^r$$

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Decimal approximations should be correct to 3 places after the decimal point.

For items 1 through 3, sketch the graph of the function. Write the equations of any asymptotes, and the coordinates of any intercepts.



1. $f(x) = \frac{3}{x^2 - 3x - 10}$



2. $f(x) = \frac{x - 4}{x^2 + 2x - 8}$



3. $f(x) = 2 + \frac{5}{x - 3}$



4. Which of the following is the equation of the horizontal asymptote of the graph of the function $f(x) = \frac{4x^2 - 2}{x^2 - 5}$

A $x = \frac{2}{5}$

B $x = 5$

C $y = \frac{1}{2}$

D $y = 4$

For items 5 and 6, solve the equations. Check for any extraneous roots.



5. $\frac{3x}{x-1} = \frac{12}{x^2-1} + 2$



6. $\frac{x}{x-2} + \frac{3x}{x-4} = \frac{32-2x}{x^2-6x+8}$



7. Which of the following statements is false?

A $f(x) = \ln x$ has a domain of all positive real numbers

B $f(x) = \ln x$ increases without bound as x increases without bound.

C The graph of $f(x) = e^{-x}$ is the graph of $g(x) = e^x$ reflected about the y -axis.

D $f(x) = e^{-x}$ has a range of all real numbers.

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8. Which of the following is equivalent to $\log_5\left(\frac{5}{x^3}\right)$?

- A $5 - 3\log_5 x$
- B $1 - 3\log_5 x$
- C $-3\log_5 x$
- D $2\log_5 x$



9. Which of the following is equivalent to $\log\left(\frac{y^2}{\sqrt{x}}\right)$?

- A $2\log y - 2\log x$
- B $2\log y - \frac{1}{2}\log x$
- C $\log(2y) - \log\left(\frac{1}{2}x\right)$
- D $\frac{\log(y^2)}{\log(\sqrt{x})}$



10. Which of the following is equivalent to $3\ln x + \ln z - \ln y$?

- A $3\ln\left(\frac{xz}{y}\right)$
- B $\ln\left(\frac{xz}{y}\right)^3$
- C $\ln\left(\frac{x^3z}{y}\right)$
- D $\ln\left(\frac{(xz)^3}{y}\right)$



11. Which of the following is equivalent to $\frac{\log 20}{\log 2}$?

- A $\log 10$
- B $\log 18$
- C $\log_2 20$
- D $\log_{20} 2$

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For items 12 and 13, evaluate. Write your answer correct to three decimal places.

12. $\log_7 6904$

13. $\log_4 0.0123$



14. The graph of $g(x)$ is the graph of $f(x) = 3^x$ reflected about the x -axis and translated 5 units to the left. What is the function rule for $g(x)$?



15. The graph of $g(x)$ is the graph of $f(x) = 9^x$ reflected about the y -axis and stretched horizontally by a factor of 7. What is the function rule for $g(x)$?

For items 16 through 23, solve. If necessary, write your answer correct to three places after the decimal point.



16. $3e^{2x} - 7 = 50$

17. $2.8^x = 345$

18. $5 \cdot 2^{3x-1} + 20 = 520$

19. $\ln(x+4) = 2$

20. $\log_7(3x-1) = 2$



21. $\log_5 x + \log_5(x-4) = 1$



22. $\log_2(x+5) - \log_2(x-1) = 2\log_2 2$



23. $2 - \log x = \log 20$

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24. A colony of insects has an initial population of 600. The number of insects triples every 4 weeks.
- Write a function for the number of insects after t weeks.
 - What will the number of insects be after 7 weeks?
 - After how many weeks will the number of insects be 12000?
25. A house has an initial value of \$100,000. Its value increases by 8% a year. Write a function that represents the value of the house after t years.
26. A population that grows continuously has an initial value of 400. After $t = 8$ years, the population is 1000. What is the continuous growth rate of this population?
27. Jack puts \$5000 in the bank at 6% interest compounded monthly. How many years will it take the amount of money in the account to become \$6000 ?
28. The population of a certain city t years after 2000 is given by the function $P(t) = 1300e^{0.04t}$.
- What was the initial population in 2000?
 - What is the rate of growth of the population?
 - What will the population be in 2010?
 - At what time t will the population be 2500?
29. The percentage of adult height obtained by a boy x years old is modeled by the function $f(x) = 29 + 49 \log(x+1)$.



- Approximately what percentage of his adult height is a 9 year-old?
- At what age will a boy reach 90% of his adult height?

For items 30 and 31, eliminate the parameter to find the relationship between x and y .



30. $x = 3t$
 $y = 6t - 7$



31. $x = 2t + 4$
 $y = 4t + 5$

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32. A golfer hits the ball from the ground at a 20° angle with an initial velocity of 132 feet per second.
- Give the motion of the ball as a set of parametric equations representing the horizontal movement of the ball after t seconds.
 - Give the location of the ball after 2 seconds.
33. A punter kicks a football from at height of 2 feet with an initial velocity of 82 feet per second at an angle of 63 degrees with the horizontal.
- Write parametric equations for the horizontal and vertical distances that the ball is from the punter after t seconds.
 - What is the position of the ball at $t = 1.5$ seconds.
 - A player 6 feet tall is standing 50 yards (150 feet) from the punter directly in the path of the kick. Will the ball go over his head? Why or why not?
 - Suppose that the kick returner lets the ball hit the ground. When will the ball hit the ground, and how far from the punter will the ball hit the ground?



34. Write the vector and parametric equations of the line that passes through the points $(1, 4)$ and $(3, 9)$.



35. Given the points $A(2, -1)$ and $B(0, 4)$. What is the vector \overline{AB} in component form?

A $\langle 2, -5 \rangle$

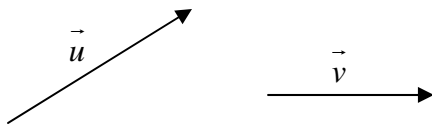
B $\langle -2, 5 \rangle$

C $\langle 2, 3 \rangle$

D $\langle -2, -3 \rangle$

36. What is the magnitude and direction of the vector $\langle -6, 8 \rangle$?

37. Look at the vectors \vec{u} and \vec{v} below



Represent the graphically the vector $\vec{u} + \vec{v}$ using both the head-to-tail method and the parallelogram method of addition.

38. Represent the vector $\vec{u} - \vec{v}$ graphically.

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For items 39 through 42, use the vectors $\vec{a} = \langle 2, -3 \rangle$, $\vec{b} = \langle 6, 5 \rangle$, and $\vec{c} = \langle r, -9 \rangle$



39. $\vec{a} \cdot \vec{b} =$



40. The angle between \vec{a} and $\vec{b} =$



41. Find the value of r such that the vectors \vec{a} and \vec{c} are parallel.



42. Find the value of r such that the vectors \vec{b} and \vec{c} are perpendicular.



43. What is the distance from the point $(2, -3)$ and the line $3x - 4y + 12 = 0$?



44. How different pizzas can be made using 3 toppings from a list of 8 available toppings?



45. How many different way are there to arrange 5 photographs on a shelf?

For items 46 and 47 represent the series using summation notation.



46. $8 + 16 + 32 + 64 + \dots$



47. $11 + 15 + 19 + 23 + 27 + 31 + 35$

For items 48 through 50, determine the sum, if any, of the series.



48. $9 + 3 + 1 + \frac{1}{3} + \dots$



49. $\sum_{n=1}^{\infty} 3 \cdot 2^n$



50. $\sum_{n=1}^{\infty} 8 \left(\frac{3}{4} \right)^n$

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For items 51 and 52, evaluate.



51. $\sum_{n=1}^4 (3n+1)$



52. $\sum_{n=5}^8 4n$

53. Expand. $(x-2)^4$



54. What is the coefficient of the x^3 term in the expansion of $(x+2)^5$?